

Greatest solutions of equations in CLL_R and its application[☆]

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Abstract

This paper explores the process calculus CLL_R furtherly. First, we prove that for any equation $X =_{RS} t_X$ such that X is strongly guarded in t_X , $\langle X | X = t_X \rangle$ is the largest solution w.r.t \sqsubseteq_{RS} . Second, we encode a fragment of action-based CTL in CLL_R .

Keywords: CLL_R , Solution of equations, Action-based CTL

1. Introduction

It is well-known that process algebra and temporal logic take different standpoint for looking at specifications and verifications of reactive and concurrent systems, and offer complementary advantages [16]. To take advantage of these two paradigms when designing systems, a few of theories for heterogeneous specifications have been proposed, e.g., [4, 5, 6, 8, 10, 11, 12, 15]. Among them, Lüttgen and Vogler propose the notion of logic labelled transition system (Logic LTS or LLTS for short), which combines operational and logical styles of specification in one unified framework [10, 11, 12]. In addition to usual process operators (e.g., CSP-style parallel composition, hiding, etc) and logic operators (disjunction and conjunction), some standard temporal logic operators, such as “always” and “unless”, are also integrated into this framework [12], which allows ones to freely mix operational and logic operators when designing systems.

Lüttgen and Vogler’s approach is entirely semantic, and doesn’t provide any kind of syntactic calculus. Recently, we propose a LLTS-oriented process calculus CLL_R , and establish the uniqueness of solutions of equations in CLL_R under a certain circumstance [17].

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This paper works on CLL_R furtherly. Our main contributions include:

(1) We will show that, without the assumption that X does not occur in the scope of any conjunction in t , the given equation $X =_{RS} t$ may have more than one consistent solution. This answers conjecture in [17] negatively. Under the hypothesis that X is strongly guarded in a given (open) term t , it is shown that the recursive process $\langle X | X = t \rangle$ is indeed the greatest (w.r.t \sqsubseteq_{RS}) consistent solution of the equation $X =_{RS} t$ whenever consistent solutions exist.

(2) We encode a temporal logic language action-based CTL [12] in CLL_R so that safety properties could be described directly without resorting to complicated settings [12], which are used to embed temporal logic operators into LLTS.

The rest of this paper is organized as follows. The calculus CLL_R and its semantics are recalled in the next section. Section 3 show that for any given equation $X =_{RS} t$ such that X is strongly guarded in t , $\langle X | X = t \rangle$ is the greatest solution w.r.t \sqsubseteq_{RS} . We encode action-based CTL in Section 4. The paper is concluded with Section 5, where a brief discussion is given.

2. Preliminaries

The purpose of this section is to fix our notation and terminology, and to introduce some concepts that underlie our work in all other parts of the paper.

2.1. Logic LTS and ready simulation

Let Act be the set of visible action names ranged over by a, b , etc., and let Act_τ denote $Act \cup \{\tau\}$ ranged over by α and β , where τ represents invisible actions. A labelled transition system with predicate is a quadruple $(P, Act_\tau, \rightarrow, F)$, where P is a set of states, $\rightarrow \subseteq P \times Act_\tau \times P$ is the transition relation and $F \subseteq P$.

As usual, we write $p \xrightarrow{\alpha}$ (or, $p \not\xrightarrow{\alpha}$) if $\exists q \in P. p \xrightarrow{\alpha} q$ ($\nexists q \in P. p \xrightarrow{\alpha} q$, resp.). The ready set $\{\alpha \in Act_\tau | p \xrightarrow{\alpha}\}$ of a given state p is denoted by $\mathcal{I}(p)$. A state p is stable if $p \not\xrightarrow{\tau}$. We also list some useful decorated transition relations:

$p \xrightarrow{\alpha}_F q$ iff $p \xrightarrow{\alpha} q$ and $p, q \notin F$;
 $p \xrightarrow{\epsilon} q$ iff $p \xrightarrow{(\tau)^*} q$, where $(\tau)^*$ is the transitive and reflexive closure of τ ;
 $p \xrightarrow{\alpha} q$ iff $\exists r, s \in P. p \xrightarrow{\epsilon} r \xrightarrow{\alpha} s \xrightarrow{\epsilon} q$;
 $p \xrightarrow{\gamma} |q$ iff $p \xrightarrow{\gamma} q \not\xrightarrow{\tau}$ with $\gamma \in Act_\tau \cup \{\epsilon\}$;
 $p \xrightarrow{\epsilon}_F q$ iff there exists a sequence of τ -transitions from p to q such that all states along this sequence, including p and q , are not in F ; the decorated transition $p \xrightarrow{\alpha}_F q$ may be defined similarly;
 $p \xrightarrow{\gamma}_F |q$ iff $p \xrightarrow{\gamma}_F q \not\xrightarrow{\tau}$ with $\gamma \in Act_\tau \cup \{\epsilon\}$.

Notice that the notation $p \xRightarrow{\gamma} |q$ in [11, 12] has the same meaning as $p \xrightarrow{\gamma}_F |q$ in this paper, while $p \xRightarrow{\gamma} q$ in this paper does not involve any requirement on F -predicate.

Definition 2.1 (Logic LTS [11]). An LTS $(P, Act_\tau, \rightarrow, F)$ is an LLTS if, for each $p \in P$,

- (LTS1) $p \in F$ if $\exists \alpha \in \mathcal{I}(p) \forall q \in P(p \xrightarrow{\alpha} q \text{ implies } q \in F)$;
(LTS2) $p \in F$ if $\nexists q \in P.p \xrightarrow{F} q$.

Moreover, an LTS $(P, Act_\tau, \rightarrow, F)$ is τ -pure if, for each $p \in P$, $p \xrightarrow{\tau}$ implies $\nexists a \in Act. p \xrightarrow{a}$.

Compared with usual LTSs, it is one distinguishing feature of LLTS that it involves consideration of inconsistencies. The main motivation behind such consideration lies in dealing with inconsistencies caused by conjunctive composition. Formally, the predicate F is used to denote the set of all inconsistent states that represent empty behaviour that cannot be implemented [12]. The condition (LTS1) formalizes the backward propagation of inconsistencies, and (LTS2) captures the intuition that divergence (i.e., infinite sequences of τ -transitions) should be viewed as catastrophic. For more intuitive ideas and motivation about inconsistency, the reader may refer [10, 11].

A variant of the usual notion of weak ready simulation [1, 9] is adopted to capture the refinement relation in [11, 12]. It has been proven that such kind of ready simulation is the largest precongruence w.r.t parallel composition and conjunction which satisfies the desired property that an inconsistent specification can only be refined by inconsistent ones (see Theorem 21 in [11]).

Definition 2.2 (Ready simulation on LLTS [11]). Let $(P, Act_\tau, \rightarrow, F)$ be a LLTS. A relation $\mathcal{R} \subseteq P \times P$ is a stable ready simulation relation, if for any $(p, q) \in \mathcal{R}$ and $a \in Act$
(RS1) both p and q are stable;
(RS2) $p \notin F$ implies $q \notin F$;
(RS3) $p \xrightarrow{a}_F p'$ implies $\exists q'. q \xrightarrow{a}_F q'$ and $(p', q') \in \mathcal{R}$;
(RS4) $p \notin F$ implies $\mathcal{I}(p) = \mathcal{I}(q)$.

We say that p is stable ready simulated by q , in symbols $p \sqsubseteq_{\sim_{RS}} q$, if there exists a stable ready simulation relation \mathcal{R} with $(p, q) \in \mathcal{R}$. Further, p is ready simulated by q , written $p \sqsubseteq_{RS} q$, if $\forall p'(p \xrightarrow{F} p' \text{ implies } \exists q'(q \xrightarrow{F} q' \text{ and } p' \sqsubseteq_{\sim_{RS}} q'))$. The kernels of $\sqsubseteq_{\sim_{RS}}$ and \sqsubseteq_{RS} are denoted by \approx_{RS} and $=_{RS}$ resp.. It is easy to see that $\sqsubseteq_{\sim_{RS}}$ itself is a stable ready simulation relation and both $\sqsubseteq_{\sim_{RS}}$ and \sqsubseteq_{RS} are pre-order.

2.2. The calculus CLL_R and its operational semantics

This subsection introduces the LLTS-oriented process calculus CLL_R presented in [17]. Let V_{AR} be an infinite set of variables. The terms of CLL_R can be given by the following BNF grammar

$$t ::= 0 \mid \perp \mid (\alpha.t) \mid (t \square t) \mid (t \wedge t) \mid (t \vee t) \mid (t \parallel_A t) \mid X \mid \langle Z | E \rangle$$

where $X \in V_{AR}$, $\alpha \in Act_\tau$, $A \subseteq Act$ and recursive specification $E = E(V)$ with $V \subseteq V_{AR}$ is a set of equations $\{X = t \mid X \in V\}$ and Z is a variable in V that acts as the initial variable.

Most of these operators are from CCS [13] and CSP [7]: 0 is the process capable of doing no action; $\alpha.t$ is action prefixing; \square is non-deterministic external choice; \parallel_A is a CSP-style parallel composition. \perp represents an inconsistent process with empty behavior. \vee and \wedge are logical operators, which are intended for describing logical combinations of processes.

For any term $\langle Z|E \rangle$ with $E = E(V)$, each variable in V is bound with scope E . This induces the notion of free occurrence of variable, bound (and free) variables and α -equivalence as usual. A term t is a *process* if it is closed, that is, it contains no free variable. The set of all processes is denoted by $T(\Sigma_{\text{CLL}_R})$. Unless noted otherwise we use p, q, r to represent processes. Throughout this paper, as usual, we assume that recursive variables are distinct from each other and no recursive variable has free occurrence; moreover we don't distinguish between α -equivalent terms and use \equiv for both syntactical identical and α -equivalence. In the sequel, we often denote $\langle X|\{X = t_X\} \rangle$ briefly by $\langle X|X = t_X \rangle$.

For any recursive specification $E(V)$ and term t , the term $\langle t|E \rangle$ is obtained from t by simultaneously replacing all free occurrences of each $X (\in V)$ by $\langle X|E \rangle$, that is, $\langle t|E \rangle \equiv t\{\langle X|E \rangle/X : X \in V\}$. For example, consider $t \equiv X \square a.\langle Y|Y = X \square Y \rangle$ and $E(\{X\}) = \{X = t_X\}$ then $\langle t|E \rangle \equiv \langle X|X = t_X \rangle \square a.\langle Y|Y = \langle X|X = t_X \rangle \square Y \rangle$. In particular, for any $E(V)$ and $t \equiv X$, $\langle t|E \rangle \equiv \langle X|E \rangle$ whenever $X \in V$ and $\langle t|E \rangle \equiv X$ if $X \notin V$.

A context $C_{\tilde{X}}$ is a term whose free variables are in some n -tuple distinct variables $\tilde{X} = (X_1, \dots, X_n)$ with $n \geq 0$. Given $\tilde{p} = (p_1, \dots, p_n)$, the term $C_{\tilde{X}}\{p_1/X_1, \dots, p_n/X_n\}$ ($C_{\tilde{X}}\{\tilde{p}/\tilde{X}\}$ for short) is obtained from $C_{\tilde{X}}$ by replacing X_i by p_i for each $i \leq n$ simultaneously. A context $C_{\tilde{X}}$ is stable if $C_{\tilde{X}}\{\tilde{0}/\tilde{X}\} \not\equiv \tilde{\tau}$.

An occurrence of X in t is strongly (or, weakly) guarded if such occurrence is within some subexpression $a.t_1$ with $a \in \text{Act}$ ($\tau.t_1$ or $t_1 \vee t_2$ resp.). A variable X is strongly (or, weakly) guarded in t if each occurrence of X is strongly (weakly resp.) guarded. A recursive specification $E(V)$ is guarded if for each $X \in V$ and $Z = t_Z \in E(V)$, each occurrence of X in t_Z is (weakly or strongly) guarded. As usual, we assume that all recursive specifications considered in the remainder of this paper are guarded. SOS rules of CLL_R are listed in Table 1, where $a \in \text{Act}$, $\alpha \in \text{Act}_\tau$ and $A \subseteq \text{Act}$. All rules are divided into two parts:

Operational rules specify behaviours of processes. Negative premises in Rules Ra_2 , Ra_3 , Ra_{13} and Ra_{14} give τ -transition precedence over visible transitions, which guarantees that the transition model of CLL_R is τ -pure. Rules Ra_9 and Ra_{10} illustrate that the operational aspect of $t_1 \vee t_2$ is same as internal choice in usual process calculus. Rule Ra_6 reflects that conjunction operator is a synchronous product for visible transitions. The operational rules of the other operators are as usual.

Predicate rules specify the inconsistency predicate F . Rule Rp_1 says that \perp is inconsistent. Hence \perp cannot be implemented. While 0 is consistent and implementable. Thus 0 and \perp represent different processes. Rule Rp_3 reflects that if both two disjunctive parts are inconsistent then so is the disjunction. Rules $Rp_4 - Rp_9$ describe the system design strategy that if one part is incon-

sistent, then so is the whole composition. Rules Rp_{10} and Rp_{11} reveal that a stable conjunction is inconsistent whenever its conjuncts have distinct ready sets. Rules Rp_{13} and Rp_{15} are used to capture (LTS2) in Def. 2.1. Intuitively, these two rules say that if all stable τ -descendants of z are inconsistent, then z itself is inconsistent.

Operational rules

$$\begin{array}{lll}
Ra_1 \frac{-}{\alpha.x_1 \xrightarrow{\alpha} x_1} & Ra_2 \frac{x_1 \xrightarrow{a} y_1, x_2 \xrightarrow{\bar{a}}}{x_1 \sqcap x_2 \xrightarrow{a} y_1} & Ra_3 \frac{x_1 \xrightarrow{\bar{a}}, x_2 \xrightarrow{a} y_2}{x_1 \sqcap x_2 \xrightarrow{a} y_2} \\
Ra_4 \frac{x_1 \xrightarrow{\tau} y_1}{x_1 \sqcap x_2 \xrightarrow{\tau} y_1 \sqcap x_2} & Ra_5 \frac{x_2 \xrightarrow{\tau} y_2}{x_1 \sqcap x_2 \xrightarrow{\tau} x_1 \sqcap y_2} & Ra_6 \frac{x_1 \xrightarrow{a} y_1, x_2 \xrightarrow{a} y_2}{x_1 \wedge x_2 \xrightarrow{a} y_1 \wedge y_2} \\
Ra_7 \frac{x_1 \xrightarrow{\tau} y_1}{x_1 \wedge x_2 \xrightarrow{\tau} y_1 \wedge x_2} & Ra_8 \frac{x_2 \xrightarrow{\tau} y_2}{x_1 \wedge x_2 \xrightarrow{\tau} x_1 \wedge y_2} & \\
Ra_9 \frac{-}{x_1 \vee x_2 \xrightarrow{\tau} x_1} & Ra_{10} \frac{-}{x_1 \vee x_2 \xrightarrow{\tau} x_2} & \\
Ra_{11} \frac{x_1 \xrightarrow{\tau} y_1}{x_1 \parallel_A x_2 \xrightarrow{\tau} y_1 \parallel_A x_2} & Ra_{12} \frac{x_2 \xrightarrow{\tau} y_2}{x_1 \parallel_A x_2 \xrightarrow{\tau} x_1 \parallel_A y_2} & \\
Ra_{13} \frac{x_1 \xrightarrow{a} y_1, x_2 \xrightarrow{\bar{a}}}{x_1 \parallel_A x_2 \xrightarrow{a} y_1 \parallel_A x_2} (a \notin A) & Ra_{14} \frac{x_1 \xrightarrow{\bar{a}}, x_2 \xrightarrow{a} y_2}{x_1 \parallel_A x_2 \xrightarrow{a} x_1 \parallel_A y_2} (a \notin A) & \\
Ra_{15} \frac{x_1 \xrightarrow{a} y_1, x_2 \xrightarrow{a} y_2}{x_1 \parallel_A x_2 \xrightarrow{a} y_1 \parallel_A y_2} (a \in A) & Ra_{16} \frac{\langle t_X | E \rangle \xrightarrow{\alpha} y}{\langle X | E \rangle \xrightarrow{\alpha} y} (X = t_X \in E) &
\end{array}$$

Predicative rules

$$\begin{array}{lll}
Rp_1 \frac{-}{\perp F} & Rp_2 \frac{x_1 F}{\alpha.x_1 F} & Rp_3 \frac{x_1 F, x_2 F}{x_1 \vee x_2 F} \\
Rp_4 \frac{x_1 F}{x_1 \sqcap x_2 F} & Rp_5 \frac{x_2 F}{x_1 \sqcap x_2 F} & Rp_6 \frac{x_1 F}{x_1 \parallel_A x_2 F} \\
Rp_7 \frac{x_2 F}{x_1 \parallel_A x_2 F} & Rp_8 \frac{x_1 F}{x_1 \wedge x_2 F} & Rp_9 \frac{x_2 F}{x_1 \wedge x_2 F} \\
Rp_{10} \frac{x_1 \xrightarrow{a} y_1, x_2 \xrightarrow{\bar{a}}, x_1 \wedge x_2 \xrightarrow{\bar{a}}}{x_1 \wedge x_2 F} & Rp_{11} \frac{x_1 \xrightarrow{\bar{a}}, x_2 \xrightarrow{a} y_2, x_1 \wedge x_2 \xrightarrow{\bar{a}}}{x_1 \wedge x_2 F} & \\
Rp_{12} \frac{x_1 \wedge x_2 \xrightarrow{\alpha} z, \{yF : x_1 \wedge x_2 \xrightarrow{\alpha} y\}}{x_1 \wedge x_2 F} & Rp_{13} \frac{\{yF : x_1 \wedge x_2 \xrightarrow{\alpha} y\}}{x_1 \wedge x_2 F} & \\
Rp_{14} \frac{\langle t_X | E \rangle F}{\langle X | E \rangle F} (X = t_X \in E) & Rp_{15} \frac{\{yF : \langle X | E \rangle \xrightarrow{\alpha} y\}}{\langle X | E \rangle F} &
\end{array}$$

Table 1: SOS rules of CLL_R

It has been shown that CLL_R has the unique stable transition model M_{CLL_R} [17], which exactly consists of all positive literals of the form $t \xrightarrow{\alpha} t'$ or tF that are provable in $Strip(CL L_R, M_{CLL_R})$. Here $Strip(CL L_R, M_{CLL_R})$ is the stripped version [2] of CLL_R w.r.t M_{CLL_R} . Each rule in $Strip(CL L_R, M_{CLL_R})$ is of the form $\frac{pprem(r)}{conc(r)}$ for some ground instance r of rules in CLL_R such that $M_{CLL_R} \models nprem(r)$, where $nprem(r)$ (or, $pprem(r)$) is the set of negative (positive resp.)

premises of r , $\text{conc}(r)$ is the conclusion of r and $M_{\text{CLL}_R} \models \text{nprem}(r)$ means that for each $t \not\rightarrow \in \text{nprem}(r)$, $t \xrightarrow{\alpha} s \notin M_{\text{CLL}_R}$ for any $s \in T(\Sigma_{\text{CLL}_R})$.

The LTS associated with CLL_R , in symbols $\text{LTS}(\text{CLL}_R)$, is the quadruple $(T(\Sigma_{\text{CLL}_R}), \text{Act}_\tau, \rightarrow_{\text{CLL}_R}, F_{\text{CLL}_R})$, where $p \xrightarrow{\alpha}_{\text{CLL}_R} p'$ iff $p \xrightarrow{\alpha} p' \in M_{\text{CLL}_R}$, and $p \in F_{\text{CLL}_R}$ iff $pF \in M_{\text{CLL}_R}$. Therefore $p \xrightarrow{\alpha}_{\text{CLL}_R} p'$ (or, $p \in F_{\text{CLL}_R}$) iff $\text{Strip}(\text{CLL}_R, M_{\text{CLL}_R}) \vdash p \xrightarrow{\alpha} p'$ (pF resp.) for any p, p' and $\alpha \in \text{Act}_\tau$. For simplification, in the following we omit the subscripts in $\xrightarrow{\alpha}_{\text{CLL}_R}$ and F_{CLL_R} .

We end this section by quoting some results from [17].

Lemma 2.3. *Let p and q be any two processes. Then*

- (1) $p \vee q \in F$ iff $p, q \in F$;
- (2) $\alpha.p \in F$ iff $p \in F$ for each $\alpha \in \text{Act}_\tau$;
- (3) $p \odot q \in F$ iff either $p \in F$ or $q \in F$ with $\odot \in \{\square, \parallel_A\}$;
- (4) $p \in F$ or $q \in F$ implies $p \wedge q \in F$;
- (5) $0 \notin F$ and $\perp \in F$;
- (6) $\langle X|E \rangle \in F$ iff $\langle t_X|E \rangle \in F$ for each X with $X = t_X \in E$.

Theorem 2.4. *$\text{LTS}(\text{CLL}_R)$ is a τ -pure LLTS. Moreover if $p \in F$ and $\tau \in \mathcal{I}(p)$ then $\forall q(p \xrightarrow{\tau} q \text{ implies } q \in F)$.*

Theorem 2.5 (precongruence). *If $p \sqsubseteq_{RS} q$ then $C_X\{p/X\} \sqsubseteq_{RS} C_X\{q/X\}$.*

3. More on solutions of equations in CLL_R

In [17], the following theorem has been obtained.

Theorem (Unique solution). *For any $p, q \notin F$ and t_X where X is strongly guarded and does not occur in the scope of any conjunction, if $p =_{RS} t_X\{p/X\}$ and $q =_{RS} t_X\{q/X\}$ then $p =_{RS} q$. Moreover $\langle X|X = t_X \rangle$ is the unique consistent solution (modulo $=_{RS}$) of the equation $X =_{RS} t_X$ whenever consistent solutions exist.*

As we know, temporal operators could be described in equational style, represented by fixpoint of some equations [3]. Such style requires us to remove the special requirement (i.e. X does not occur in the scope of any conjunction) occurring in Theorem Unique Solution. In the following, we give a negative answer for this removal by providing a counterexample:

Observation 3.1. Consider the equation $X = t_X$ where $t_X \equiv (\langle Y|Y = a.Y \rangle \wedge a.X) \vee (\langle Z|Z = b.Z \rangle \wedge b.X)$. In the following, we show that $\langle X|X = a.X \rangle$ is a consistent solution of this equation. First we show that $\langle X|X = a.X \rangle \notin F$. Contrarily, assume that $\langle X|X = a.X \rangle \in F$. Then the last rule applied in the proof tree of $\text{Strip}(\text{CLL}_R, M_{\text{CLL}_R}) \vdash \langle X|X = a.X \rangle F$ is

$$\frac{a.\langle X|X = a.X \rangle F}{\langle X|X = a.X \rangle F} \text{ or } \frac{\{rF : \langle X|X = a.X \rangle \xrightarrow{\epsilon} |r\}}{\langle X|X = a.X \rangle F}.$$

It is not difficult to see that every proof tree of $\langle X | X = a.X \rangle F$ has proper subtree with root $\langle X | X = a.X \rangle F$, this contradicts the well-foundedness of proof tree, as desired. Second we show that $\langle X | X = a.X \rangle$ indeed is a solution of $X =_{RS} t_X$. Clearly, due to Rules Rp_{10} and Rp_{11} , $\langle Z | Z = b.Z \rangle \wedge \langle X | X = a.X \rangle \in F$, which is the unique b -derivative of $\langle Z | Z = b.Z \rangle \wedge b.\langle X | X = a.X \rangle$. Hence $\langle Z | Z = b.Z \rangle \wedge b.\langle X | X = a.X \rangle \in F$ by Condition (LTS1) in Def. 2.1 and Theorem 2.4. Moreover we also have $\langle X | X = a.X \rangle =_{RS} \langle Y | Y = a.Y \rangle \wedge a.\langle X | X = a.X \rangle$. Therefore $\langle X | X = a.X \rangle =_{RS} t_X \{ \langle X | X = a.X \rangle / X \}$. Similarly, $\langle X | X = b.X \rangle$ is another consistent solution. However, $\langle X | X = a.X \rangle \neq_{RS} \langle X | X = b.X \rangle$.

In the remainder of this section, we intend to show that the recursive process $\langle X | X = t \rangle$ captures the extreme solution of the equation $X = t$. To this end, a number of results in [17] are listed below.

Lemma 3.2. *If $C_X \{p/X\} \xrightarrow{\tau} r$ then*

- (1) *either there exists C'_X such that $r \equiv C'_X \{p/X\}$ and $C_X \{q/X\} \xrightarrow{\tau} C'_X \{q/X\}$ for any q ,*
- (2) *or there exist $C'_{X,Z}$ and p' such that $p \xrightarrow{\tau} p'$, $r \equiv C'_{X,Z} \{p/X, p'/X\}$ and $C_X \{q/X\} \xrightarrow{\tau} C'_{X,Z} \{q/X, q'/Z\}$ for any $q \xrightarrow{\tau} q'$.*

Lemma 3.3. *Let $a \in \text{Act}$. If $C_X \{p/X\} \xrightarrow{a} r$ then there exists $C'_{X,\tilde{Y}}$ such that*

- (1) *$r \equiv C'_{X,\tilde{Y}} \{p/X, \widetilde{p'_Y/\tilde{Y}}\}$ for some $\widetilde{p'_Y}$ with $p \xrightarrow{a} p'_Y$ for each $Y \in \tilde{Y}$, and*
- (2) *if $C_X \{q/X\}$ is stable and for each $Y \in \tilde{Y}$, $q \xrightarrow{a} q'_Y$, then $C_X \{q/X\} \xrightarrow{a} C'_{X,\tilde{Y}} \{q/X, \widetilde{q'_Y/\tilde{Y}}\}$.*

Lemma 3.4. *Let X be guarded in C_X . If $C_X \{p/X\} \xrightarrow{\alpha} r$ then there exists B_X such that $r \equiv B_X \{p/X\}$ and $C_X \{q/X\} \xrightarrow{\alpha} B_X \{q/X\}$ for any q .*

Lemma 3.5. *If $C_X \{p/X\} \xrightarrow{\varepsilon} |r$ then there exist stable $C'_{X,\tilde{Y}}$ and stable p'_Y for each $Y \in \tilde{Y}$ such that (1) $p \xrightarrow{\tau} |p'_Y$ for each $Y \in \tilde{Y}$ and $r \equiv C'_{X,\tilde{Y}} \{p/X, \widetilde{p'_Y/\tilde{Y}}\}$; (2) for any q such that $q \xrightarrow{\tau} \text{iff } p \xrightarrow{\tau}$, if $q \xrightarrow{\tau} |q'_Y$ for each $Y \in \tilde{Y}$ then $C_X \{q/X\} \xrightarrow{\varepsilon} |C'_{X,\tilde{Y}} \{q/X, \widetilde{q'_Y/\tilde{Y}}\}$; (3) if X is strongly guarded in C_X then so it is in $C'_{X,\tilde{Y}}$ and $\tilde{Y} = \emptyset$.*

Before giving the main result of this section, we prove a lemma concerning F -predicate.

Lemma 3.6. *If X is strongly guarded in t_X and $p \sqsubseteq_{RS} t_X \{p/X\}$ then for any C_Y , $C_Y \{t_X \{p/X\}/Y\} \notin F$ implies $C_Y \{\langle X | X = t_X \rangle / Y\} \notin F$.*

Proof. Clearly, by Lemmas 3.2, 3.3 and 3.4, we get

$$C_Y \{t_X \{p/X\}/Y\} \xrightarrow{\alpha} \text{iff } C_Y \{\langle X | X = t_X \rangle / Y\} \xrightarrow{\alpha} \text{ for any } C_Y. \quad (3.6.1)$$

Set $\Omega \triangleq \{B_Y \{\langle X | X = t_X \rangle / Y\} : B_Y \{t_X \{p/X\}/Y\} \notin F\}$. Clearly, it suffice to prove that $F \cap \Omega = \emptyset$. Conversely, suppose that $F \cap \Omega \neq \emptyset$. Due to the well-foundedness of proof trees, to complete the proof, it is sufficient to show that,

for each $C_Y\{\langle X|X = t_X\rangle/Y\} \in \Omega$, any proof tree for $Strip(\text{CLL}, M_{\text{CLL}_R}) \vdash C_Y\{\langle X|X = t_X\rangle/Y\}F$ has a proper subtree with root sF for some $s \in \Omega$. We shall prove this as follows. Let \mathcal{T} be any proof tree of $C_Y\{\langle X|X = t_X\rangle/Y\}F$. It is a routine case analysis based on the last rule applied in \mathcal{T} . We treat only non-trivial three cases and leave the others to the reader.

Case 1. $C_Y \equiv Y$.

Then $C_Y\{\langle X|X = t_X\rangle/Y\} \equiv \langle X|X = t_X\rangle$. So the last rule applied in \mathcal{T} is $\frac{\langle t_X|X=t_X\rangle F}{\langle X|X=t_X\rangle F}$ or $\frac{\{rF:\langle X|X=t_X\rangle \xrightarrow{\tau} |r\}}{\langle X|X=t_X\rangle F}$.

For the former, since $C_Y\{t_X\{p/X\}/Y\} \equiv t_X\{p/X\} \notin F$ and $p \sqsubseteq_{RS} t_X\{p/X\}$, we have $t_X\{t_X\{p/X\}/X\} \notin F$ due to Theorem 2.5. Hence $\langle t_X|X = t_X\rangle \equiv t_X\{\langle X|X = t_X\rangle/X\} \in \Omega$. For the latter, we treat the non-trivial subcase that $\langle X|X = t_X\rangle \xrightarrow{\tau}$. Since $t_X\{p/X\} \notin F$, $t_X\{p/X\} \xrightarrow{\tau} |s$ for some s . For this transition, since X is strongly guarded in t_X , by Lemma 3.5, there exists a stable t'_X with strongly guarded X such that $s \equiv t'_X\{p/X\}$ and $t_X\{\langle X|X = t_X\rangle\} \xrightarrow{\tau} t'_X\{\langle X|X = t_X\rangle/X\}$. Further, by Lemma 3.4, $\langle X|X = t_X\rangle \xrightarrow{\tau} |t'_X\{\langle X|X = t_X\rangle/X\}$ due to $\langle X|X = t_X\rangle \xrightarrow{\tau}$. Moreover $t'_X\{t_X\{p/X\}/X\} \notin F$ because of $s \equiv t'_X\{p/X\} \notin F$ and $p \sqsubseteq_{RS} t_X\{p/X\}$. Hence $t'_X\{\langle X|X = t_X\rangle/X\} \in \Omega$, as desired.

Case 2. $C_Y \equiv \langle Z|E\rangle$.

The last rule applied in \mathcal{T} is one of following two cases: $\frac{\langle t_Z|E\rangle\{\langle X|X=t_X\rangle/Y\}F}{\langle Z|E\rangle\{\langle X|X=t_X\rangle/Y\}F}$ or $\frac{\{rF:\langle Z|E\rangle\{\langle X|X=t_X\rangle/Y\} \xrightarrow{\tau} |r\}}{\langle Z|E\rangle\{\langle X|X=t_X\rangle/Y\}F}$.

By Lemma 2.3(6), the former is easy to handle and omitted. Next we treat the latter. Since $C_Y\{t_X\{p/X\}/Y\} \notin F$, $C_Y\{t_X\{p/X\}/Y\} \xrightarrow{\tau} |s$ for some s . For this transition, by Lemma 3.5, there exist stable $C'_{Y,\widetilde{W}}$ and $\widetilde{s'_W}$ such that $s \equiv C'_{Y,\widetilde{W}}\{t_X\{p/X\}, \widetilde{s'_W}/\widetilde{W}\}$ and $t_X\{p/X\} \xrightarrow{\tau} |s'_W$ for each $W \in \widetilde{W}$. Further, for each $t_X\{p/X\} \xrightarrow{\tau} |s'_W$, there exists stable t'^W_X with strongly guarded X such that $s'_W \equiv t'^W_X\{p/X\}$ and $t_X\{\langle X|X = t_X\rangle/X\} \xrightarrow{\tau} t'^W_X\{\langle X|X = t_X\rangle/X\}$. So, by Lemma 3.4, $\langle X|X = t_X\rangle \xrightarrow{\tau} |t'^W_X\{\langle X|X = t_X\rangle/X\}$ for each $W \in \widetilde{W}$ and hence $C_Y\{\langle X|X = t_X\rangle/Y\} \xrightarrow{\tau} |C'_{Y,\widetilde{W}}\{\langle X|X = t_X\rangle/Y, t'^W_X\{\langle X|X = t_X\rangle/X\}/\widetilde{W}\} \equiv u$. Since $s \equiv C'_{Y,\widetilde{W}}\{t_X\{p/X\}/Y, t'^W_X\{t_X\{p/X\}/X\}/\widetilde{W}\} \notin F$ and $p \sqsubseteq_{RS} t_X\{p/X\}$, we get $C'_{Y,\widetilde{W}}\{t_X\{p/X\}/Y, t'^W_X\{t_X\{p/X\}/X\}/\widetilde{W}\} \notin F$, which implies $u \in \Omega$, as desired.

Case 3. $C_Y \equiv B_Y \wedge D_Y$.

We split the argument into the following four subcases.

Case 3.1. $\frac{B_Y\{\langle X|X=t_X\rangle/Y\}F}{C_Y\{\langle X|X=t_X\rangle/Y\}F}$.

Since $C_Y\{t_X\{p/X\}/Y\} \notin F$, $B_Y\{t_X\{p/X\}/Y\} \notin F$ by Lemma 2.3. So, $B_Y\{\langle X|X = t_X\rangle/Y\} \notin F$, as desired.

Case 3.2. $\frac{B_Y\{\langle X|X=t_X\rangle/Y\} \xrightarrow{\alpha}}{C_Y\{\langle X|X=t_X\rangle/Y\} F}$ with $D_Y\{\langle X|X=t_X\rangle/Y\} \not\xrightarrow{q}$ and $C_Y\{\langle X|X=t_X\rangle/Y\} \not\xrightarrow{r}$.

By (3.6.1), a contradiction arises due to $C_Y\{t_X\{p/X\}/Y\} \notin F$.

Case 3.3. $\frac{\{rF:C_Y\{\langle X|X=t_X\rangle/Y\} \xrightarrow{\alpha} r\}}{C_Y\{\langle X|X=t_X\rangle/Y\} F}$.

Similar to the second case of Case 2, omitted.

Case 3.4. $\frac{C_Y\{\langle X|X=t_X\rangle/Y\} \xrightarrow{\alpha} r', \{rF:C_Y\{\langle X|X=t_X\rangle/Y\} \xrightarrow{\alpha} r\}}{C_Y\{\langle X|X=t_X\rangle/Y\} F}$.

Then $C_Y\{\langle X|X=t_X\rangle/Y\} \xrightarrow{\alpha} r'$. Since $C_Y\{t_X\{p/X\}/Y\} \notin F$, by (3.6.1),

$$C_Y\{t_X\{p/X\}/Y\} \xrightarrow{\alpha}_F s \text{ for some } s. \quad (3.6.2)$$

In the following, we treat two cases based on α .

Case 3.4.1. $\alpha = \tau$.

For (3.6.2), by Lemma 3.2, either $s \equiv C'_Y\{t_X\{p/X\}/Y\}$ for some C'_Y such that $C_Y\{q/Y\} \xrightarrow{\tau} C'_Y\{q/Y\}$ for any q , or there exist s' and $C'_{Y,Z}$ such that $s \equiv C'_{Y,Z}\{t_X\{p/X\}/Y, s'/Z\}$ and $t_X\{p/X\} \xrightarrow{\tau} s'$. For the former, it is trivial. Next we treat the later. For $t_X\{p/X\} \xrightarrow{\tau} s'$, since X is strongly guarded in t_X , by Lemma 3.4, there exists t'_X such that $s' \equiv t'_X\{p/X\}$ and $t_X\{\langle X|X=t_X\rangle/X\} \xrightarrow{\tau} t'_X\{\langle X|X=t_X\rangle/X\}$. Then $\langle X|X=t_X\rangle \xrightarrow{\tau} t'_X\{\langle X|X=t_X\rangle/X\}$ and hence $C_Y\{\langle X|X=t_X\rangle/Y\} \xrightarrow{\tau} C'_{Y,Z}\{\langle X|X=t_X\rangle/Y, t'_X\{\langle X|X=t_X\rangle/X\}/Z\} \equiv u$. Since $p \sqsubseteq_{RS} t_X\{p/X\}$ and $s \equiv C'_{Y,Z}\{t_X\{p/X\}/Y, t'_X\{p/X\}/Z\} \notin F$, we get $C'_{Y,Z}\{t_X\{p/X\}/Y, t'_X\{t_X\{p/X\}/X\}/Z\} \notin F$. Clearly, $u \in \Omega$, as desired.

Case 3.4.2. $\alpha \in Act$.

For (3.6.2), by Lemma 3.3, $s \equiv C'_{Y,\tilde{Z}}\{t_X\{p/X\}, \widetilde{s'_Z/\tilde{Z}}\}$ for some $C'_{Y,\tilde{Z}}$ and $\widetilde{s'_Z}$ such that $t_X\{p/X\} \xrightarrow{\alpha} s'_Z$ for each $Z \in \tilde{Z}$. Since X is strongly guarded in t_X , for each $t_X\{p/X\} \xrightarrow{\alpha} s'_Z$, by Lemma 3.4, there exists t'^Z_X such that $s'_Z \equiv t'^Z_X\{p/X\}$ and $t_X\{\langle X|X=t_X\rangle/X\} \xrightarrow{\alpha} t'^Z_X\{\langle X|X=t_X\rangle/X\}$. Then $\langle X|X=t_X\rangle \xrightarrow{\alpha} t'^Z_X\{\langle X|X=t_X\rangle/X\}$ for each $Z \in \tilde{Z}$ and hence $C_Y\{\langle X|X=t_X\rangle/Y\} \xrightarrow{\alpha} C'_{Y,\tilde{Z}}\{\langle X|X=t_X\rangle/Y, \widetilde{t'^Z_X\{\langle X|X=t_X\rangle/X\}/\tilde{Z}}\} \equiv u$ by (3.6.1). Since $p \sqsubseteq_{RS} t_X\{p/X\}$ and $s \equiv C'_{Y,\tilde{Z}}\{t_X\{p/X\}, \widetilde{t'^Z_X\{p/X\}/\tilde{Z}}\} \notin F$, by Theorem 2.5, we get $C'_{Y,\tilde{Z}}\{t_X\{p/X\}, \widetilde{t'^Z_X\{t_X\{p/X\}/X\}/\tilde{Z}}\} \notin F$. Clearly, $u \in \Omega$, as desired. \square

Next we recall an equivalent formulation of \sqsubseteq_{RS} and an up-to technique.

Definition 3.7. A relation $\mathcal{R} \subseteq T(\Sigma_{\text{CLL}_R}) \times T(\Sigma_{\text{CLL}_R})$ is an alternative ready simulation relation, if for any $(p, q) \in \mathcal{R}$ and $a \in Act$

(RSi) $p \xrightarrow{\epsilon}_F |p'$ implies $\exists q'. q \xrightarrow{\epsilon}_F |q'$ and $(p', q') \in \mathcal{R}$;

(RSiii) $p \xrightarrow{a}_F |p'$ and p, q stable implies $\exists q'. q \xrightarrow{a}_F |q'$ and $(p', q') \in \mathcal{R}$;

(RSiv) $p \notin F$ and p, q stable implies $\mathcal{I}(p) = \mathcal{I}(q)$.

We write $p \sqsubseteq_{ALT} q$ if there exists an alternative ready simulation relation \mathcal{R} with $(p, q) \in \mathcal{R}$.

Definition 3.8 (ALT up to $\sqsubseteq_{\sim RS}$). A relation $\mathcal{R} \subseteq T(\Sigma_{\text{CLL}_R}) \times T(\Sigma_{\text{CLL}_R})$ is an alternative ready simulation relation up to $\sqsubseteq_{\sim RS}$, if for any $(p, q) \in \mathcal{R}$ and

$a \in \text{Act}$

(ALT-upto-1) $p \xrightarrow{a}_F |p'$ implies $\exists q'. q \xrightarrow{a}_F |q'$ and $p' \sqsubseteq_{\sim RS} \mathcal{R} \sqsubseteq_{\sim RS} q'$;

(ALT-upto-2) $p \xrightarrow{a}_F |p'$ and p, q stable implies $\exists q'. q \xrightarrow{a}_F |q'$ and $p' \sqsubseteq_{\sim RS} \mathcal{R} \sqsubseteq_{\sim RS} q'$;

(ALT-upto-3) $p \notin F$ and p, q stable implies $\mathcal{I}(p) = \mathcal{I}(q)$.

It has been proved that $\sqsubseteq_{RS} = \sqsubseteq_{ALT}$ [11] and if \mathcal{R} is an alternative ready simulation relation up to $\sqsubseteq_{\sim RS}$, then $\mathcal{R} \subseteq \sqsubseteq_{RS}$ [17]. With these results, we could prove the next lemma.

Lemma 3.9. *Let X be strongly guarded in t_X . If $p \sqsubseteq_{RS} t_X\{p/X\}$ then $t_X\{p/X\} \sqsubseteq_{RS} \langle X|X = t_X \rangle$.*

Proof. Set $\mathcal{R} \triangleq \{(B_Y\{t_X\{p/X\}/Y\}, B_Y\{\langle X|X = t_X \rangle/Y\})\}$. It is sufficient to prove that \mathcal{R} is an alternative ready simulation relation up to $\sqsubseteq_{\sim RS}$. Let $(C_Y\{t_X\{p/X\}/Y\}, C_Y\{\langle X|X = t_X \rangle/Y\}) \in \mathcal{R}$. By Lemma 3.2, 3.3 and 3.4, (ALT-upto-3) holds clearly. Next we handle the other two clauses.

(ALT-upto-1) Assume $C_Y\{t_X\{p/X\}/Y\} \xrightarrow{a}_F |s$. For this transition, by Lemma 3.5, $s \equiv C'_{Y, \tilde{Z}}\{t_X\{p/X\}/Y, \widetilde{s'_Z/\tilde{Z}}\}$ for some stable $C'_{Y, \tilde{Z}}$ and $\widetilde{s'_Z}$ such that $t_X\{p/X\} \xrightarrow{a} |s'_Z$ for each $Z \in \tilde{Z}$. Further, for each $t_X\{p/X\} \xrightarrow{a} |s'_Z$, since X is strongly guarded in t_X , there exists stable $t'_X{}^Z$ with strongly guarded X such that $s'_Z \equiv t'_X{}^Z\{p/X\}$ and $t_X\{\langle X|X = t_X \rangle/X\} \xrightarrow{a} |t'_X{}^Z\{\langle X|X = t_X \rangle/X\}$. So $\langle X|X = t_X \rangle \xrightarrow{a} |t'_X{}^Z\{\langle X|X = t_X \rangle/X\}$ for each $Z \in \tilde{Z}$ and hence $C_Y\{\langle X|X = t_X \rangle/Y\} \xrightarrow{a} |C'_{Y, \tilde{Z}}\{\langle X|X = t_X \rangle/Y, \widetilde{t'_X{}^Z\{\langle X|X = t_X \rangle/X\}/\tilde{Z}}\} \equiv u$. Since $s \equiv C'_{Y, \tilde{Z}}\{t_X\{p/X\}/Y, \widetilde{t'_X{}^Z\{p/X\}/\tilde{Z}}\} \notin F$ and $p \sqsubseteq_{RS} t_X\{p/X\}$, by Lemma 3.4 and Theorem 2.5, we obtain $s \sqsubseteq_{\sim RS} C'_{Y, \tilde{Z}}\{t_X\{p/X\}/Y, \widetilde{t'_X{}^Z\{t_X\{p/X\}/X\}/\tilde{Z}}\} \notin F$, which implies $u \notin F$ by Lemma 3.6. Clearly $C_Y\{\langle X|X = t_X \rangle/Y\} \xrightarrow{a}_F |u$ by Lemma 2.4, and $s \sqsubseteq_{\sim RS} \mathcal{R}u$, as desired.

(ALT-upto-2) Assume that $C_Y\{t_X\{p/X\}/Y\}$ and $C_Y\{\langle X|X = t_X \rangle/Y\}$ are stable and $C_Y\{t_X\{p/X\}/Y\} \xrightarrow{a}_F |s$. Then $C_Y\{t_X\{p/X\}/Y\} \xrightarrow{a}_F r \xrightarrow{a}_F |s$ for some r . For the a -transition, by Lemma 3.3, $r \equiv C'_{Y, \tilde{Z}}\{t_X\{p/X\}/Y, \widetilde{r'_Z/\tilde{Z}}\}$ for some $C'_{Y, \tilde{Z}}$ and $\widetilde{r'_Z}$ such that $t_X\{p/X\} \xrightarrow{a} r'_Z$ for each $Z \in \tilde{Z}$. Since X is strongly guarded in t_X , for each $t_X\{p/X\} \xrightarrow{a} r'_Z$, by Lemma 3.4, there exists $t'_X{}^Z$ such that $r'_Z \equiv t'_X{}^Z\{p/X\}$ and $t_X\{\langle X|X = t_X \rangle/X\} \xrightarrow{a} |t'_X{}^Z\{\langle X|X = t_X \rangle/X\}$. Then $\langle X|X = t_X \rangle \xrightarrow{a} |t'_X{}^Z\{\langle X|X = t_X \rangle/X\}$ for each $Z \in \tilde{Z}$ and hence $C_Y\{\langle X|X = t_X \rangle/Y\} \xrightarrow{a} C'_{Y, \tilde{Z}}\{\langle X|X = t_X \rangle/Y, \widetilde{t'_X{}^Z\{\langle X|X = t_X \rangle/X\}/\tilde{Z}}\} \equiv v$. Let $u \equiv C'_{Y, \tilde{Z}}\{t_X\{p/X\}/Y, \widetilde{t'_X{}^Z\{t_X\{p/X\}/X\}/\tilde{Z}}\}$. Since $p \sqsubseteq_{RS} t_X\{p/X\}$, by

Theorem 2.5, we have $r \equiv C'_{Y, \tilde{Z}} \{t_X \{p/X\}/Y, t'_X \{p/X\}/\tilde{Z}\} \sqsubseteq_{RS} u$. Hence since $r \xRightarrow{F} s$, we have $u \xRightarrow{F} t$ and $s \sqsubseteq_{\sim_{RS}} t$ for some t . Since $u \mathcal{R} v$, by (ALT-upto-1), $v \xRightarrow{F} t'$ for some t' such that $t \sqsubseteq_{\sim_{RS}} \mathcal{R} \sqsubseteq_{\sim_{RS}} t'$. Therefore, by Lemma 3.6, $C_Y \{\langle X|X = t_X \rangle/Y\} \xRightarrow{a} t'$ and $s \sqsubseteq_{\sim_{RS}} t \sqsubseteq_{\sim_{RS}} \mathcal{R} \sqsubseteq_{\sim_{RS}} t'$. \square

Now with the previous lemma, it is not difficult to get

Theorem 3.10. *For any equation $X =_{RS} t_X$ such that X is strongly guarded in t_X , if consistent solution exists then $\langle X|X = t_X \rangle$ is the greatest consistent solution.*

4. Encoding ACTL in CLL_R

In [12], Lüttgen and Vogler introduce a fragment of action-based CTL [14] (ACTL for short), embed it into LLTS and present the desired compatibility result between logical satisfaction and \sqsubseteq_{RS} . In this section, we recall their ACTL and encode it in CLL_R under the hypothesis that Act is finite.

Definition 4.1. The action-based CTL is defined by BNF:

$$\phi ::= tt \mid ff \mid en(a) \mid dis(a) \mid \phi \vee \phi \mid \phi \wedge \phi \mid [a]\phi \mid \mathcal{A}\phi \mid \phi \mathcal{W} \phi$$

where $a \in Act$. $T(\Sigma_{ACTL})$ denotes the set of all terms in ACTL.

$en(a)$ and $dis(a)$ denote enabledness and disabledness of action a resp. $[a]$, \mathcal{A} and \mathcal{W} are usual *next*, *always* and *weak until* operators. For more motivations and intuitions about these operators, the reader may refer to [12].

Before encoding formulas of ACTL in CLL_R , we introduce some useful notations. Given n terms $t_i (0 \leq i \leq n-1)$ in $T(\Sigma_{CLL_R})$, the general external choice $\sqcap_{i < n} t_i$ and disjunction $\bigvee_{i < n} t_i$ are defined recursively as:

$$\begin{aligned} \sqcap_{i < 0} t_i &\triangleq 0, \sqcap_{i < 1} t_i \triangleq t_0, \text{ and } \sqcap_{i < k+1} t_i \triangleq (\sqcap_{i < k} t_i) \sqcap t_k \text{ for } k \geq 1; \\ \bigvee_{i < 1} t_i &\triangleq t_0, \text{ and } \bigvee_{i < k+1} t_i \triangleq (\bigvee_{i < k} t_i) \vee t_k \text{ for } k \geq 1. \end{aligned}$$

The general conjunction $\bigwedge_{i < n} t_i$ is defines similarly as disjunction.

Given a term ϕ in $T(\Sigma_{ACTL})$, the encoding of ϕ , denoted by $\mathcal{E}(\phi)$, is defined as:

$$\begin{aligned} \mathcal{E}(tt) &\triangleq \langle X|X = \bigvee_{A \subseteq Act} \sqcap_{a \in A} a.X \rangle & \mathcal{E}(ff) &\triangleq \perp \\ \mathcal{E}(en(a)) &\triangleq \bigvee_{a \in A \subseteq Act} \sqcap_{b \in A} b.\mathcal{E}(tt) & \mathcal{E}(dis(a)) &\triangleq \bigvee_{a \notin A \subseteq Act} \sqcap_{b \in A} b.\mathcal{E}(tt) \\ \mathcal{E}([a]\phi) &\triangleq [a](\mathcal{E}(\phi)) & \mathcal{E}(\phi_1 \vee \phi_2) &\triangleq \mathcal{E}(\phi_1) \vee \mathcal{E}(\phi_2) \\ \mathcal{E}(\phi_1 \wedge \phi_2) &\triangleq \mathcal{E}(\phi_1) \wedge \mathcal{E}(\phi_2) & \mathcal{E}(\mathcal{A}\phi) &\triangleq \langle X|X = \mathcal{E}(\phi) \wedge (\bigwedge_{a \in Act} [a]X) \rangle \\ \mathcal{E}(\phi_1 \mathcal{W} \phi_2) &\triangleq \langle X|X = \mathcal{E}(\phi_2) \vee (\mathcal{E}(\phi_1) \wedge (\bigwedge_{a \in Act} [a](X))) \rangle \end{aligned}$$

where $\lceil a \rceil \triangleq \lambda x. (\bigvee_{a \in A \subseteq Act} ((\bigwedge_{b \in A - \{a\}} b.\mathcal{E}(tt)) \Box a.x)) \vee (\bigvee_{a \notin A \subseteq Act} (\bigwedge_{b \in A} b.\mathcal{E}(tt)))$, intuitively, $\lceil a \rceil$ says “along a -transition, it is necessary that ...”.

Therefore, if we want to check a specification $p \in T(\Sigma_{CLL_R})$ satisfies some desired property $\phi \in T(\Sigma_{ACTL})$, we only check whether $p \sqsubseteq_{RS} \mathcal{E}(\phi)$ or $p \wedge \mathcal{E}(\phi) =_{RS} \perp$ holds.

Theorem 4.2. $p \models \phi$ iff $p \sqsubseteq_{RS} \mathcal{E}(\phi)$.

5. Conclusions and discussion

This paper works on LLTS-oriented process calculus CLL_R furtherly. We show that for any given equation $X =_{RS} t$ such that X is strongly guarded in t , $\langle X | X = t \rangle$ is the largest consistent solution w.r.t \sqsubseteq_{RS} if consistent solutions exist. Moreover we also encode a temporal logic language ACTL in CLL_R .

For further work, it is very interesting to study the structure of the solution space $\{p : p \sqsubseteq_{RS} t_X\{p/X\}\}$ if X is strongly guarded in t_X .

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